Question

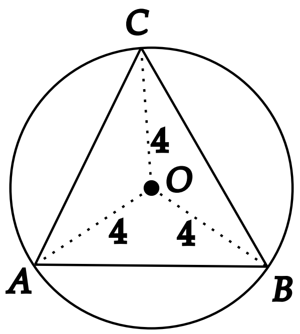
What is the area of an inscribed equilateral triangle in a circle of radius 4?

Round your answer to the nearest tenth.

20.8

Solution

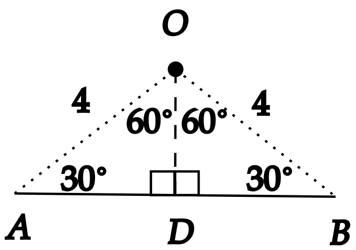
Consider the below diagram:



We have an equilateral triangle ABC that is inscribed inside a circle of radius 4. OA, OB and OC represent the radii of the circle.

The first step is to recognise that triangles AOB, BOC and AOC are all congruent by virtue of OA = OB = OC (radii) and AC = AB = BC (as ABC is equilateral). Then, the problem reduces to finding the area of AOB and multiplying its result by 3. Also notice that because of the congruency, $\angle AOB = \angle BOC = \angle AOC = 120°$.

To find AOB, we can take one of two approaches. The first one involves taking the perpendicular through O, intersecting AB at a point we'll call D. Given that AOB is isosceles, this means that the perpendicular also serves as a bisector of AB:



We have a pair of 30-60-90 triangles. As a result, we can show that OD = 2 and AD = $\sqrt{12} = 2\sqrt{3}$. Hence, the area of triangle AOB is $\frac{1}{2}bh = \frac{1}{2}\left(2 \times 2\sqrt{3} \times 2\right) = 4\sqrt{3}$.

As noted above, the area of triangle ACB is three times the area of AOB, which is $3 \times 4\sqrt{3} = 12\sqrt{3}$. This is 20.8 to one decimal place.

A shortcut using trigonometry

This is out of scope for the GRE and is included for those interested.

There is a quicker way to find the area of ΔAOB. Recall that the area of a triangle is $\frac{1}{2}ab\sin C$. *C* in this case would be 120 degrees, and *a* = *b* = 4. Hence, the area is

$$A = \frac{1}{2} \times 4 \times 4 \times \sin 120^\circ$$
$$A = 8\sin 60^\circ$$
$$A = \frac{8(\sqrt{3})}{2}$$
$$A = 4\sqrt{3}$$

This saves you from having to divide the triangle into two halves.